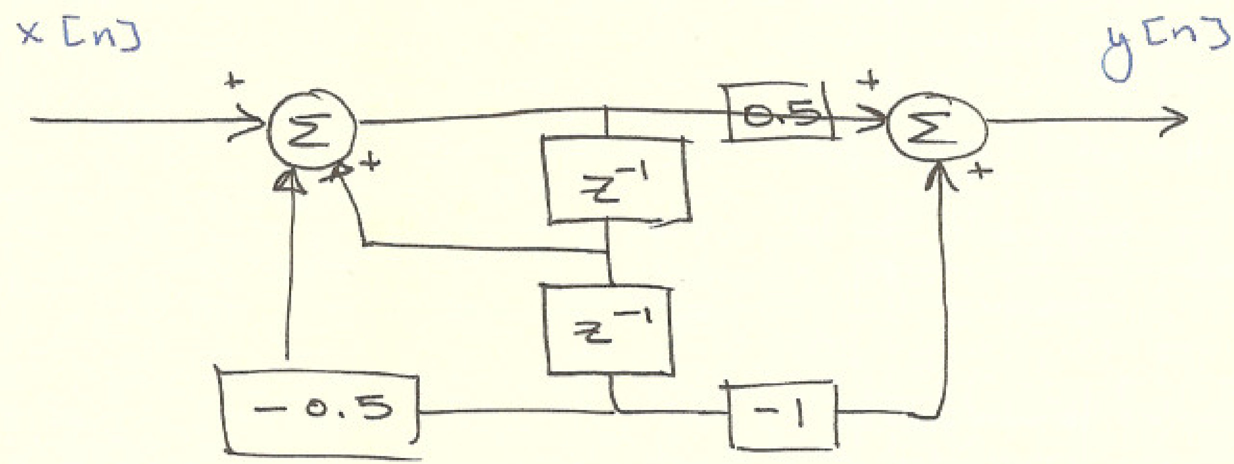
EX: Implement the following system using D-II implementation technique.

$$2y[n] - 2y[n-1] + y[n-2] = x[n] - 2x[n-2]$$

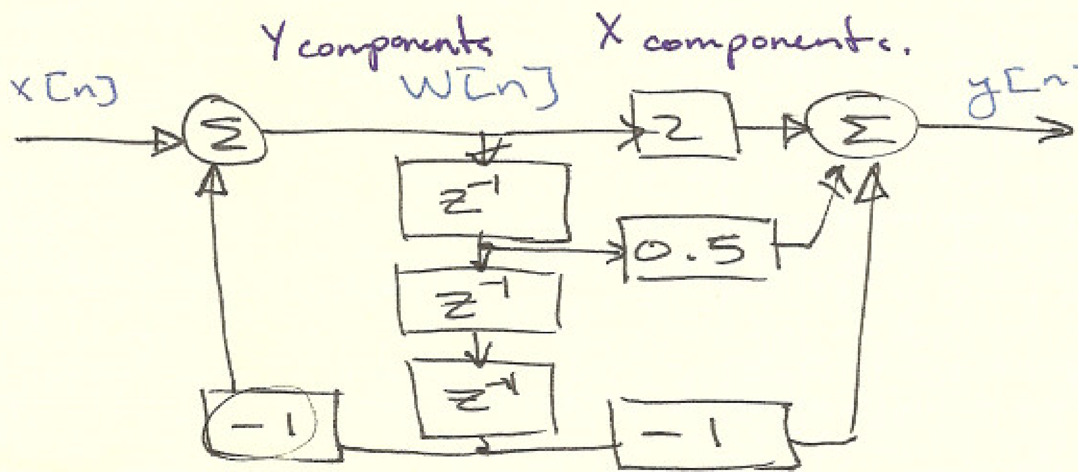
SOL:

$$y[n] = 0.5\,x[n] - x[n-2] + y[n-1] - 0.5\,y[n-2]$$



★ Be prepared to modify block diagram. Also we prepared to extract a D.E. given a block diagram.

EX:



SOL:

$$w[n] = x[n] - x[n-3]$$

$$y[n] = 2w[n] + 0.5w[n] - w[n-3]$$

$$\frac{Y(z)}{X(z)} = \ldots$$

Or solving directly from the D-II block diagram.

$$y[n] = 2x[n] + 0.5x[n-1] - x[n-3] - y[n-3]$$

FIR system (Non-recursive system)

$$y[n] = b_0 x[n] + b_2 x[n-2] + b_3 x[n-3]$$

Here only the D-I system is realisable.

Discrete Fourier Transform (DFT)

(chapter 8 from text)

FS → FT → DTFT → DFT

DSF → DFT,

DFT is a sequence rather then continuous function of a variable. It is an alternative repersentation of a finite length sequence. DFT is nessisary for some DSP algorithm (eg. FFT) exists for DFT computation

Discrete Fourier Series.

Consider a periodic sequence $x[n]$ with fundamental period N

$$x[n] = x[n + iN]$$

$$i \in \mathbb{Z}$$

It is possible to represent such a periodic sequence with a series of cosine and sine terms, or an exponential sequence of the form

$$e_k[n] = e^{j\frac{2\pi}{N}kn}$$

when k is the harmonic order.

Now

$$e_k[n] = e_{k+N}[n] = e_k[n+N]$$

Since

$$e^{j2\pi kn} = 1$$

Therefore there are N such exponential terms and hence the fourier series representation is given by,

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

very similiar to analog exponential fourier

Using the notation,

$$W_N = e^{-j\frac{2\pi}{N}}$$

then,

$$\boxed{\begin{aligned} X[k] &= \sum_{n=0}^{N-1} x[n]\, W_N^{kn} \\ x[n] &= \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{-kn} \end{aligned}}$$

These are the equations we will use the most.

EX: Find the DFS of the following periodic sequence

$$x[n] = \sum_{r=-\infty}^{\infty} \delta[n - rN]$$

SOL:

$$r \in \mathbb{Z}$$

$$= \begin{cases} 1, & n = rN \\ 0, & \text{elsewhere.} \end{cases}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} = 1$$

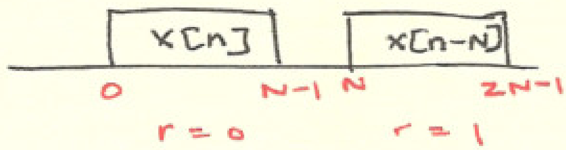
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} W_N^{-kn}$$

* Do not expect questions of discrete fourier series, it is just to lead us into discrete fourier transform.

DFT

A finite length sequence x[n] with length N can relate to a periodic sequence as...

$$x[n] = \sum_{r=-\infty}^{\infty} x[n-rN] \qquad (1)$$

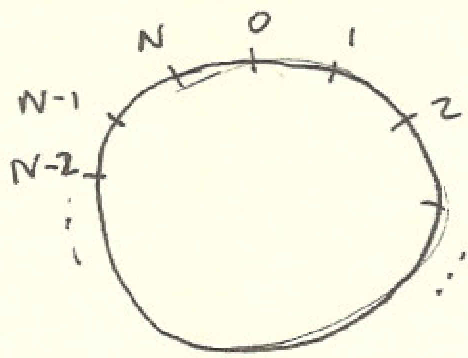


$$x[n] = \begin{cases} \tilde{x}[n], & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

Since x[n] has finite length N, there is no overlap between the terms x[n-rN]. Thus equ 1. can alternatively be written as:

$$x[n] = x[n \text{ modulus } N]$$

This means that it is a circular sequence x[N] with N point.



Note: $x[n \text{ modulous } N]$ is sometimes represented as $x[((n))N]$

Since the DFS representation of a finite length sequence is called DFT, so the DFT $X[k]$ is related to the DFS coefficient $\tilde{X}[k]$ by,

$$X[k] = \begin{cases} \tilde{X}[k] & , 0 \le k \le N-1 \\ 0 & , \text{elsewhere.} \end{cases}$$

And

$$\tilde{X}[k] = x[n \text{ modulous } N]$$

$\therefore$ N point DFT,

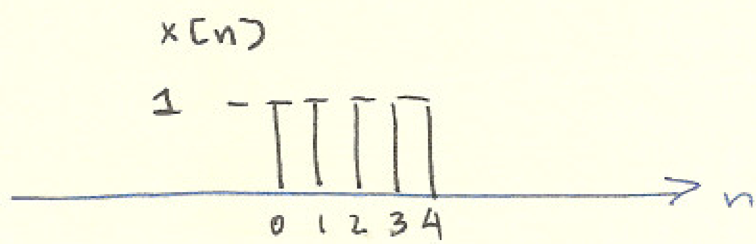
DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \qquad \boxed{0 \le k \le N-1}$$

DFT^{-1}

$$x[n] = \frac{1}{N} \sum_{n=0}^{N-1} X[k] W_N^{-kn} \qquad \boxed{0 \le n \le N-1}$$

EX:



This sequence can be considered any length greater or equal to 5.
Calculate the 5 point DFT.

SOL:

$$X[k] = \sum_{n=0}^{4} W_5^{kn}$$

5 for 5 point

$$W_5 = e^{-j\frac{2\pi}{5}}$$

$$x[k] = \frac{1 - e^{-j2\pi k}}{1 - e^{-j\frac{2\pi}{5}k}}$$

Note: All multiplications of 5, we must apply l'hopital's theorem.

$\therefore$ $x[0] = 5$

$x[1] = x[2] = x[3] = x[4] = 0$